TE and TM Modes in Circularly Shielded Slot Waveguides

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Abstract—**Cutoff wavenumbers and the field of TE and TM modes are evaluated in circularly shielded, single- or double-slot waveguides in case of infinitely thin fins. The formulation, based on field equivalence principles, is exact and leads to Carleman-type singular integral or integro-differential equations for the equivalent surface magnetic current across the slot(s). The solution of these equations is based on Neumann's expansion of the Hankel kernels and leads to numerically stable and efficient algorithms regardless of the slot widths. Numerical results for the cutoff wavenumbers, both TE and TM, are presented. By lowering the cutoff frequencies of the TE modes and by raising the corresponding ones of the TM modes considerable increase in the operating frequency bandwidth may be achieved after suitably selecting the various geometrical parameters of the configurations.**

I. Introduction

In a recent publication [1] propagation in circularly shielded, single- or double-strip lines was investigated. The dual problems of circularly shielded, single or double (coplanar) slot-lines with infinitely thin fins are the subject of the present paper.

Longitudinal slots may be used to implement directional couplers for exchanging energy between waveguides and, also, to control the modal properties by lowering or elevating the cutoff frequencies of several modes (e.g., in ridged waveguides and fin-lines [2]–[20]).

The conformability of slot- and strip-configurations with many microwave integrated circuit (MIC) devices has made the former quite attractive for use in the design of such devices. For the analysis of such guiding structures a variety of techniques has been used in the past, the great majority based on numerical approaches such as moment methods, finite differences, etc. As noted in [2]–[3] and for reasons extensively discussed in [1] an exact analytical treatment turns out to be, in general, quite a formidable task.

The analysis of circularly shielded slot (or strip) lines is an exception to this rule. The boundary value problem for such structures can be formulated conveniently in terms of Carleman-type singular integral or integrodifferential equations (S.I.E. or S.ID.E.). The kernel of these equations is a Hankel function $H_0^{(2)}(k_c|\overline{\rho}-\overline{\rho}'|)$, or some combination of such functions, for which strongly convergent series expansions are available. These expansions, based on Neumann's formula for the Hankel function [21] lead to very efficient and stable, recently developed, numerical algorithms [22]–[24] for the cutoff wavenumbers and the field components, both for narrow and wide slots (or strips) [1]. A further advantage of this approach is the fact that, for cylindrical shields, the isolation of the singular (logarithmic) term of the kernel from its analytic part leads to simple additional terms in the field expressions; in contrast, this is not possible with rectangular shields. The isolation of the logarithmic term is instrumental in applying Carleman's inversion formula for the solution of the S.I.E. and S.ID.E.

In addition to its analytical convenience, the present and other related configurations are important from the standpoint of applications as well. Thus, e.g., they may be used in building ultra-bandwidth microwave circuit elements (such as hybrid junctions, directional couplers, and polarization-selective couplers) as described in [4]. In comparison with rectangularly shielded slot lines their advantages (in addition to easy fabrication and compatibility of the dominant mode with the TE_{11} circular guide mode) are twofold [5]: (a) they provide a better control of field polarization, (which may be useful in a variety of applications involving phase shifters, travelling-wave isolators, antenna feeds, etc.; (b) due to their better attenuation characteristics they provide a potential alternative in the cases where, owing to increasing attenuation, rectangular fin lines become impractical.

The $\exp[j(\omega t - \beta z)]$ time- and z -dependence (β being the propagation constant), assumed for all field quantities, is suppressed throughout the following analysis.

II. Formulation and Solution of the Propagation Problem in Shielded Slot Waveguides

A. TE Modes in Single-Slot Waveguides

The waveguide configuration is shown in Fig. 1. The radius of the cylindrical shield is denoted by α whereas the slot of width $2w$ may be eccentrically placed at $(y = 0, h - w \leq x \leq h + w)$ along the z-axis, h being the eccentricity (this implies that the following analysis is restricted to infinitely thin fins). The guide is filled with a homogeneous dielectric characterized by the scalars (ϵ, μ).

Invoking field equivalence principles [25] the boundary value problem may be most conveniently formulated in terms of equivalent surface magnetic currents, as illustrated in Fig. 2. The latter, defined by $\overline{\boldsymbol{M}}(x) = \overline{\boldsymbol{E}}(x, 0) \times \hat{y}$, are considered to act in the absence of the slot (i.e., with the slot short-circuited).

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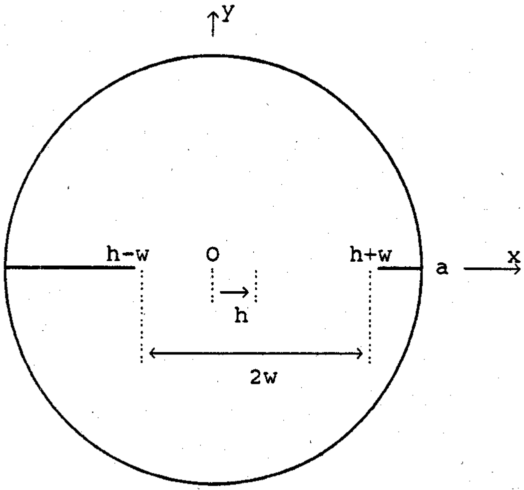

Fig. 1. The geometry of an eccentric single slot-line.

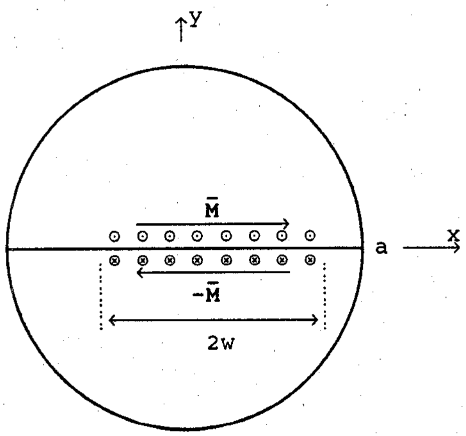

Fig. 2. The equivalent problem.

The magnetic-type Green's function $G_m(\overline{\rho}, \overline{\rho}')$ inside a semi-cylindrical shield is identified with the longitudinal component of the magnetic field excited at $\overline{\rho}(\rho, \phi)$ due to a magnetic line source $\overline{\boldsymbol{M}}_a = \hat{z} M_a \delta(\overline{\rho} - \overline{\rho}')$ impressed at $\overline{\rho}'(\rho', \phi')$. Using separation of variables one may find that [26]:

$$G_m(\overline{\rho}, \overline{\rho}') = -\frac{k_c^2}{4\omega\mu} M_a \cdot \Bigg\{ [H_0^{(2)}(k_c R^-) + H_0^{(2)}(k_c R^+)] - 2 \sum_{m=0}^{\infty} A_m(a)\, J_m(k_c \rho') \cdot \cos(m\phi') J_m(k_c \rho) \cos(m\phi)] \Bigg\}, \quad (1)$$

$$A_m(a) = \epsilon_m H_m^{(2)'}(k_c a) / J_m'(k_c a), \quad \epsilon_m = 2 - \delta_{m0} \quad (2)$$

$$R^{\pm} = \sqrt{(x - x')^2 + (y \pm y')^2}, \quad k_c^2 = k^2 - \beta^2 \quad (3)$$

k_c being the cutoff wavenumber of the propagating mode and $k = \omega\sqrt{\epsilon\mu}$.

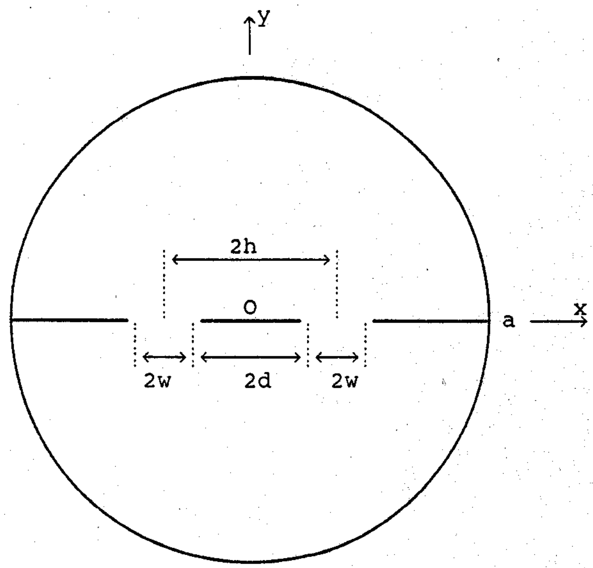

Fig. 3. The geometry of a symmetrical coplanar slot-line.

Referring to the equivalent problems shown in Fig. 2 and invoking twice the reciprocity theorem: $\langle \overline{\boldsymbol{M}}_a, \overline{\boldsymbol{M}} \rangle = \langle \overline{\boldsymbol{M}}, \overline{\boldsymbol{M}}_a \rangle\, (y' > 0), \langle \overline{\boldsymbol{M}}_a, -\overline{\boldsymbol{M}} \rangle = \langle -\overline{\boldsymbol{M}}, \overline{\boldsymbol{M}}_a \rangle\, (y' < 0)$ we are able to obtain:

$$M_a H_z(x', y') = \operatorname{sgn}(y') \int_C M(x)\, G_m(x, 0; x', y')\, dx \quad (4)$$

with C denoting the x-axis interval $[h - w, h + w]$ and $M(x) = E_x(x, 0)$. Application of the continuity condition for the magnetic field across the slot leads to the following (Carleman-type) S.I.E.:

$$\mathcal{L}\{C; M(x); x'\} \equiv \int_C M(x)\, G_m(x, 0; x', 0)\, dx = 0; \quad x' \in C. \quad (5)$$

Changing variables $x = h + wt, x' = h + wt'$ $(-1 \leq t, t' \leq 1)$, substituting in (5) and using (1) yields:

$$\int_{-1}^{1} M(t)\, H_0^{(2)}(k_c w |t - t'|)\, dt - \sum_{m=0}^{\infty} A_m(a)\, J_m[k_c(h + wt')] \cdot \int_{-1}^{1} M(t)\, J_m[k_c(h + wt)]\, dt = 0, \qquad |t'| \leq 1 \quad (6)$$

where $M(t) \equiv M[x(t)]$.

The S.I.E. (6) can be discretized as outlined in [22]–[23]. To this end we first expand $M(t)$ into the following series of Chebyshev polynomials of the first kind:

$$M(t) = (1 - t^2)^{-1/2} \sum_{N=0}^{\infty} \alpha_N T_N(t), \quad t \in [-1, 1] \quad (7)$$

in compliance with the edge condition. After substituting from (7) we multiply both sides of (6) by $T_M(t)/(1-t^2)^{1/2}$ and integrate from $t=-1$ to $t=1$. This leads to the linear algebraic system:

$$\sum_{N=0}^{\infty} \alpha_N R_{MN}(k_c w) = 0; \qquad M = 0,1,2,\cdots \tag{8}$$

where:

$$\begin{aligned} R_{MN}(k_c w) = &-j\frac{2}{\pi}[K^s_{MN}(k_c w) \\ &+ K^r_{MN}(k_c w)] - \frac{1}{4}\sum_{m=0}^{\infty} f_m(a) \end{aligned} \tag{9a}$$

$$\begin{aligned} f_m(a) &= A_m(a)\, S_N(m)\, S_M(m); \\ S_N(m) &= \sum_{\substack{p=0 \\ N+p=\text{even}}}^{\infty} \epsilon_p I(N,p)\,\Gamma(m,p); \\ \Gamma(m,p) &= J_{m-p}(k_c h) \\ &\quad + (-1)^p J_{m+p}(k_c h) \end{aligned} \tag{9b}$$

$$I(p,q) = \int_{-1}^{1} \frac{T_p(t)}{\sqrt{1-t^2}} J_q(k_c w t)\, dt = \begin{cases} \pi J_{(p+q)/2}\left(\dfrac{k_c w}{2}\right) \cdot J_{(q-p)/2}\left(\dfrac{k_c w}{2}\right); & p+q \text{ even} \\ 0; \quad p+q \text{ odd}. \end{cases} \tag{10}$$

The expression

$$\begin{aligned} -j\frac{2}{\pi}&[K^s_{MN}(k_c w) + K^r_{MN}(k_c w)] \\ &= \int_{-1}^{1}\int_{-1}^{1} \frac{T_M(t)}{\sqrt{1-t^2}} \frac{T_N(t')}{\sqrt{1-t'^2}} \\ &\quad \cdot H_0^{(2)}(k_c w|t-t'|)\, dt\, dt' \end{aligned} \tag{11}$$

which appears in (9a), has been evaluated analytically in [22]–[23] in terms of a strongly convergent, compact and numerically stable expression, in which only terms of the form $I(p,q)$ are involved. To get (8) from (6) use was made, also, of the expansions:

$$\begin{aligned} J_m&[k_c(h+wt)] \\ &= \frac{1}{2}\sum_{p=0}^{\infty} \epsilon_p J_p(k_c w t)\,\Gamma(m,p); \end{aligned}$$

$$J_q(k_c w t) = \frac{1}{\pi}\sum_{p=0}^{\infty} \epsilon_p I(p,q)\, T_p(t). \tag{12}$$

For symmetrically placed slots $(h=0), f_m(a)$ in (9b) is reduced to the single term:

$$f_m(a) = 4A_m(a)\, I(N,m)\, I(M,m). \tag{13}$$

The series over m in (9a) is, also, strongly (exponentially) convergent. From (8) by truncation and inversion, the normalized coefficients $\alpha_N/\alpha_0\,(N=1,2,\cdots)$ may be evaluated if desired. Also, from the same truncated homogeneous system, the particular values of k_c for which $\det[R_{MN}]=0$ provide the cutoff wavenumbers. The successive truncation sizes $M \times M, (M+1)\times(M+1),\cdots$ lead to sequences of convergent values for the cutoff wavenumbers.

For a thorough discussion of the efficiency and economy of the algorithm one may refer to [1], [23]–[24].

B. TE Modes in Double-Slot Waveguides

We now refer to the two-equal-slot configuration of Fig. 3 and denote by $M^{(1)}(x_1)$ and $M^{(2)}(x_2)$ the equivalent surface magnetic currents on the slots 1 and 2; they satisfy the 2×2 system of S.I.E.:

$$\begin{aligned} &\mathcal{L}\{C_1; M^{(1)}(x_1); x'\} \\ &\quad + \mathcal{L}\{C_2; M^{(2)}(x_2); x'\} = 0 \\ &\qquad (x' \in C_1 \equiv [h-w, h+w] \\ &\qquad \text{or } x' \in C_2 \equiv [-h-w, -h+w]). \end{aligned} \tag{14}$$

For (equal and) symmetrically placed slots: $M^{(1)}(x) = sM^{(2)}(-x) = M(x)$, with $s=1$ implying codirectional magnetic currents and $s=-1$ contradirectional ones. In this case the following single S.I.E. may be written as [1]:

$$\begin{aligned} &\int_{-1}^{1} M(t')\, H_0^{(2)}(k_c w|t-t'|)\, dt' \\ &\quad + s\int_{-1}^{1} M(-t'')\, H_0^{(2)} \\ &\quad \cdot [k_c|2h + w(t-t'')|]\, dt'' \\ &= \sum_{m=0}^{\infty} A_m(a)\, J_m(k_c(h+wt)) \\ &\quad \cdot \left[\int_{-1}^{1} M(t')\, J_m[k_c(h+wt')]\, dt' \right. \\ &\quad \left. + s\int_{-1}^{1} M(-t'')\, J_m[k_c(-h+wt'')]\, dt''\right]; \\ &\quad |t| \le 1 \end{aligned} \tag{15}$$

where $M(t') = M[x_1(t')]$, after changing variables: $x_1 = h+wt', x_2 = -h+wt, x' = h+wt\,(x' \in C_1)$.

The procedure outlined in the preceding subsection can be applied to (15), using for $M(t)$ the expansion (7), with the help of (12) as well as [1, (16)]. Referring to [1, Section 3] for more details, the final result is again the infinite set of equations (8), where now its matrix elements are

$$\begin{aligned} R_{MN}&(k_c w) \\ &= -j\frac{2}{\pi}[K^s_{MN}(k_c w) + K^r_{MN}(k_c w)) \\ &\quad - \frac{1}{4}\sum_{m=0}^{\infty}[1+s(-1)^m] f_m(a) \\ &\quad + \frac{1}{2}(-1)^N s\sum_{k=0}^{\infty} \epsilon_k (-1)^k H_k(2k_c h) \end{aligned}$$

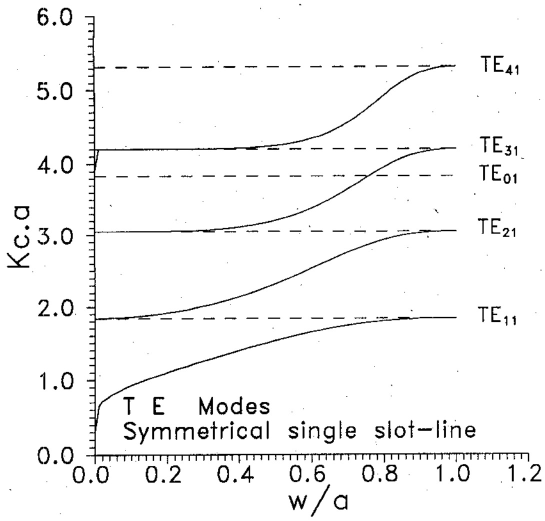

Fig. 4. Cutoff wavenumbers versus w/a of the first five even TE modes for the symmetrical single slot-line of Fig. 1 ($h = 0$).

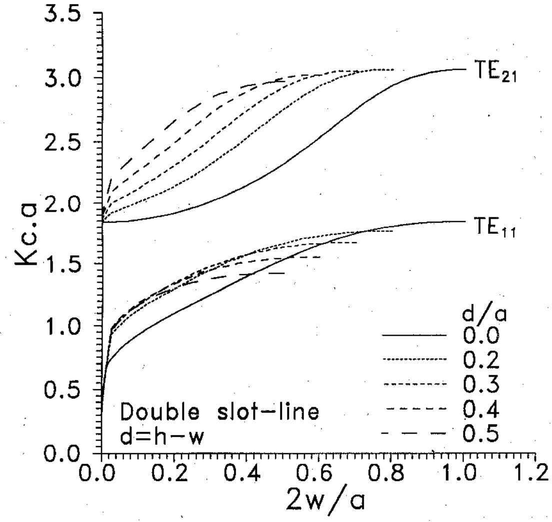

Fig. 6. Cutoff wavenumbers versus $2w/a$ for the double slot-line of Fig. 3 and for different d/a values (TE_{11} and TE_{21} modes).

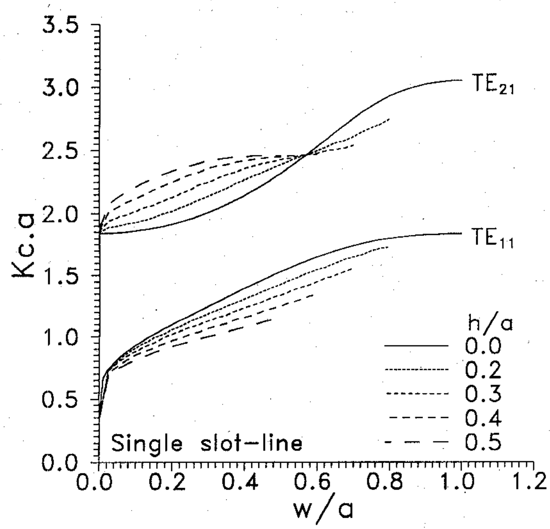

Fig. 5. Cutoff wavenumbers versus w/a for the single slot-line of Fig. 1 and for different h/a values (TE_{11} and TE_{21} modes).

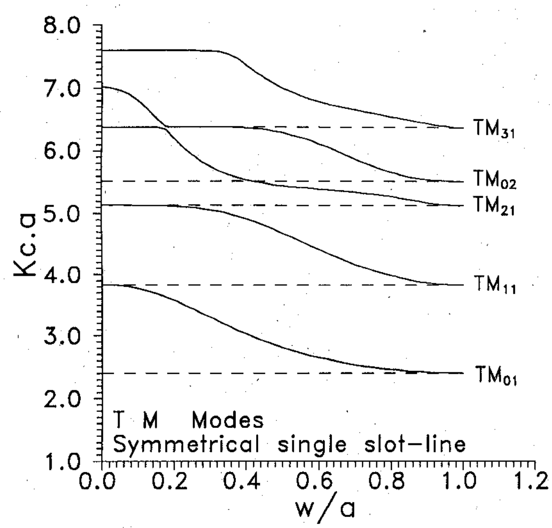

Fig. 7. Cutoff wavenumbers versus w/a for the first five even TM modes for the single slot-line of Fig. 1 ($h/a = 0$).

$$\cdot \left\{ \sum_{n=0}^{\infty} \epsilon_n I(N,n) \cdot (I(M,k+n) + (-1)^n I(M,k-n)) \right\}. \quad (16)$$

Finally, we note that for $s = 0$ we get the expression (9a) for a single slot.

C. TM Modes in Single-Slot Waveguides

For a propagating TM mode, the equivalent surface magnetic current is $\overline{\boldsymbol{M}}(x) = \hat{z}M_z(x) + \hat{x}M_x(x); M_z(x) = E_x(x,0), M_x(x) = -E_z(x,0)$. The electric-type Green's function $G_e(\overline{\rho}, \overline{\rho}')$ inside a semi-cylindrical shield is identified with the longitudinal component of the electric field excited at $\overline{\rho}$ due to an electric line source $\overline{J}_a = \hat{z}I_a\delta(\overline{\rho} - \overline{\rho}')$ impressed at $\overline{\rho}'$. Using separation of variables one may find that [26]:

$$G_e(\overline{\rho}, \overline{\rho}') = -\frac{k_c^2}{4\omega\epsilon} I_a \cdot \left\{ [H_0^{(2)}(k_c R^-) - H_0^{(2)}(k_c R^+)] - 2\sum_{m=1}^{\infty} B_m(a)\, J_m(k_c\rho') \cdot \sin(m\phi') J_m(k_c\rho) \sin(m\phi) \right\} \quad (17)$$

$$B_m(a) = 2H_m^{(2)}(k_c a)/J_m(k_c a) \quad (18)$$

with $R^{\pm}$ defined in (3).

Referring to the equivalent problems of Fig. 2 and using again twice the reciprocity theorem: $\langle \overline{\boldsymbol{J}}_a, \overline{\boldsymbol{M}} \rangle = \langle \overline{\boldsymbol{M}}, \overline{\boldsymbol{J}}_a \rangle$ ($y' >$

0), $\langle \overline{\boldsymbol{J}}_a, -\overline{\boldsymbol{M}}\rangle = \langle -\overline{\boldsymbol{M}}, \overline{\boldsymbol{J}}_a\rangle$ $(y' < 0)$ one gets:

$$I_a E_z(x', y') = -\operatorname{sgn}(y') \frac{\omega\epsilon}{jk_c^2} \cdot \int_C M_x(x) \frac{\partial}{\partial y} G_e(x, 0; x', y')\, dx. \quad (19)$$

Using (19) and (17) and expressing $H_x(x', y')$ in terms of $E_z(x', y')$ yields the following S.ID.E.:

$$\begin{aligned} &\pounds\{C; M_x(x); x'\} \\ &\equiv \left(\frac{d^2}{dx'^2} + k_c^2\right) \int_C M_x(x) H_0^{(2)} \\ &\cdot (k_c|x - x'|)\, dx - \frac{k_c^2}{4} \sum_{m=1}^{\infty} B_m(a) \\ &\cdot \Theta(m, k_c x') \int_C M_x(x)\, \Theta(m, k_c x)\, dx = 0; \\ &x' \in C; \quad \Theta(m, b) = J_{m-1}(b) + J_{m+1}(b) \quad (20) \end{aligned}$$

after applying the continuity condition for H_x across the slot.

Changing variables $x = h + wt, x' = h + wt'$ $(-1 \leq t, t' \leq 1)$ and expanding $M_x(t) = M_x[x(t)]$ into the following series of Chebyshev polynomials of the second kind:

$$M_x(t) = \sqrt{1 - t^2} \cdot \sum_{N=0}^{\infty} b_N U_N(t), \qquad t \in [-1, 1] \quad (21)$$

in conformity with the edge condition, the S.ID.E. (20) can be discretized as outlined in [22]–[23]. To this end, we substitute from (21) into (20), multiply both its sides by $\sqrt{1 - t'^2} U_M(t')$ and integrate from $t' = -1$ to $t' = 1$. The final result is the following linear algebraic system:

$$\sum_{N=0}^{\infty} b_N \tilde{R}_{MN}(k_c w) = 0; \qquad M = 0, 1, 2, \cdots (22)$$

$$\begin{aligned} &\tilde{R}_{MN}(k_c w) \\ &= [\tilde{K}_{MN}^s(k_c w) + \tilde{K}_{MN}^r(k_c w)] \\ &- \left(\frac{k_c w}{4}\right)^2 \sum_{m=1}^{\infty} g_m(a) \quad (23) \end{aligned}$$

$$\begin{aligned} g_m(a) &= B_m(a)\, \tilde{S}_N(m)\, \tilde{S}_M(m); \\ \tilde{S}_N(m) &= \sum_{\substack{q=0 \\ q+N=\text{even}}}^{\infty} \epsilon_q J(N, q)\, \Delta(m, q); \\ \Delta(m, q) &= \Gamma(m-1, q) + \Gamma(m+1, q) \quad (24) \\ J(p, q) &= \int_{-1}^{1} \sqrt{1 - t^2} U_p(t)\, J_q(k_c w t)\, dt \\ &= \frac{1}{2}[I(p, q) - I(p+2, q)] \quad (25) \end{aligned}$$

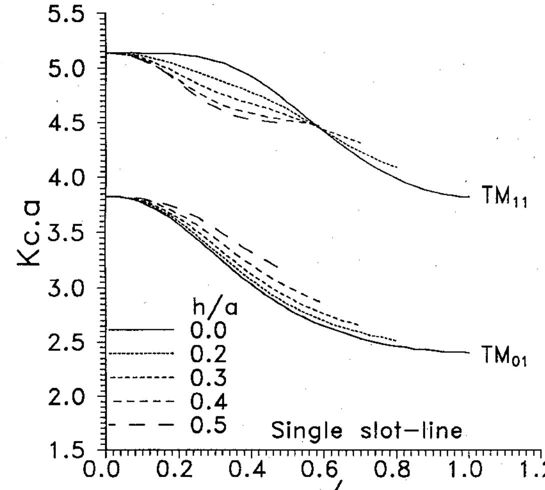

Fig. 8. Cutoff wavenumbers versus w/a for the single slot-line of Fig. 1 and for different h/a values (TM_{01} and TM_{11} modes).

with $\Gamma(m, n)$ and $I(m, n)$ defined in (9b), (10). In (23) the term:

$$\begin{aligned} &\frac{1}{w}[\tilde{K}_{MN}^s(k_c w) + \tilde{K}_{MN}^r(k_c w)] \\ &= w \int_{-1}^{1} \sqrt{1 - t'^2} U_M(t') \\ &\cdot \left[k_c^2 + \frac{1}{w^2} \frac{d^2}{dt'^2}\right] \int_{-1}^{1} \sqrt{1 - t^2}. \\ &\cdot U_N(t)\, H_0^{(2)}(k_c w|t - t'|)\, dt'\, dt \quad (26) \end{aligned}$$

assumes the analytical and, numerically, very efficient and stable expression given by [23, (25)] and [23, (26)] (see, also, [1, Appendix B]).

For symmetrically placed slots, (24) assumes the following much simpler (single term) expression:

$$\begin{aligned} g_m(a) = 4B_m &\cdot (a)\{J(N, m-1) + J(N, m+1)\} \\ &\cdot \{J(M, m-1) + J(M, m+1)\}. \quad (27) \end{aligned}$$

By truncation of (22), characteristic equation $\det[\tilde{R}_{MN}(k_c w)] = 0$, which provides the cutoff wavenumbers, leads, as before, to rapidly convergent sequences of values as the truncation size $M \times M$ increases.

D. TM Modes in Double-Slot Waveguides

For equal and symmetrically placed slots, as in Fig. 3, the case of TM modes can be formulated as in (14) using operator $\pounds$ in place of $\mathcal{L}$ and with the $\hat{x}$-components $M_x^{(1)}(x_1) =$

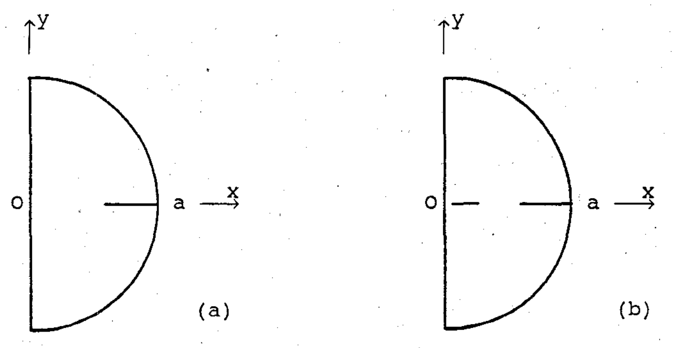

Fig. 9. Geometry of: (a) single ridged or (b) double ridged semi-circular guides.

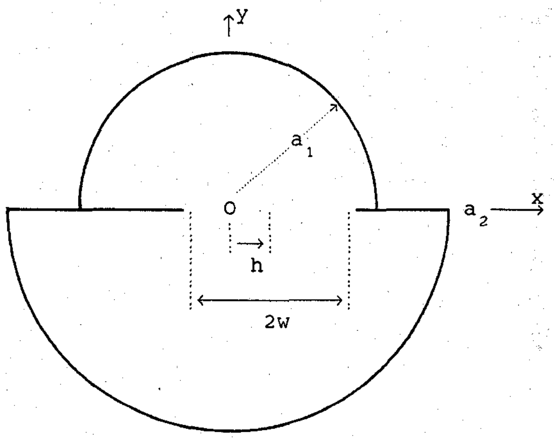

Fig. 10. Geometry of two coupled semicircular guides.

$sM_x^{(2)}(x_2) = M(x)$ of the equivalent surface magnetic current on the slots replacing $M^{(1)}(x_1) = sM^{(2)}(x_2) = M(x)$ in (14). Changing variables $(x' = h + wt, x_1 = h + wt'; x_1 \in C_1, x_2 = -h + wt; x_2 \in C_2)$, this system reduces to the following single S.ID.E.:

$$\left(\frac{d^2}{dt^2} + (k_c w)^2\right) \cdot \left[\int_{-1}^{1} M(t')\, H_0^{(2)}(k_c w|t - t'|)\, dt' + s\int_{-1}^{1} M(-t'') \cdot H_0^{(2)}[k_c|2h + w(t - t)|]\, dt''\right] - \frac{(k_c w)^2}{4}\sum_{m=1}^{\infty} B_m(a)\, \Theta(m, k_c(h + wt)) \cdot \left[\int_{-1}^{1} M(t')\, \Theta(m, k_c(h + wt'))\, dt' + s\int_{-1}^{1} M(-t'')\, \Theta(m, k_c(-h + wt''))\, dt''\right] = 0. \tag{28}$$

Using again the expansion (21) for $M(t)$ and following the procedure outlined in the preceding single-slot TM-case (see [1, Section 5] for more details), we arrive at the system (22) where now:

$$\tilde{R}_{MN}(k_c w) = [\tilde{K}_{MN}^{s}(k_c w) + \tilde{K}_{MN}^{r}(k_c w)] - \left(\frac{k_c w}{4}\right)^2 \sum_{m=1}^{\infty} [1 - s(-1)^m] g_m(a) + \frac{1}{2}(-1)^N s(k_c w)^2 \cdot \sum_{k=0}^{\infty} \epsilon_k (-1)^k H_k(2k_c h) \cdot \sum_{n=0}^{\infty} \epsilon_n J(N, n) \cdot (D(M, k + n) + (-1)^n D(M, k - n)) \tag{29}$$

$$D(M, q) = \frac{1}{4}[J(M, q - 2) + 2J(M, q) + J(M, q + 2)] \tag{30}$$

with $J(p, q)$ defined in (25).

Setting $s = 0$ in (29) we obtain (23) for the single slot.

III. Numerical Results and Discussion

The modes supported by the structures of Figs. 1 and 3 may roughly be divided [7] into fin-sensitive (TE^{fin}, TM^{fin}) and fin-insensitive (TE^{empty}, TM^{empty}) modes. The latter include the even TE_{0n} as well as all odd TE and TM empty circular guide modes. In the following we will restrict ourselves to the study of the fin-sensitive modes only.

The normalized cutoff wavenumbers $k_c a$ for the five consecutive, lower order TE^{fin} modes of the symmetrical slot line of Fig. 1 are shown in Fig. 4 as a (monotonically increasing) function of the normalized slot width w/a. As seen from these plots, the cutoff wavenumbers for all modes, as $w/a \to 1$, approach with remarkable accuracy the known values for the corresponding modes in the empty circular guide (dotted straight lines), providing a first reliability test for our approach. This justifies the classification of the modes and their characterization as TE_{mn}. In the limit, as $w/a \to 0$, they tend to their corresponding values of a semicircular guide, as expected. An important exception is the dominant TE_{11} mode whose cutoff frequency tends to zero for small values of w/a. This behavior, which may be used conveniently for lowering the admissible operating frequencies, is explained in [7]. More specifically, the limit $w/a \to 0$ may be approached by either of the following two alternatives: (a) keeping a finite and letting $w \to 0$; in this case the electric field concentration between fins, causes an equivalent capacitive loading of the guide, which tends to lower its cutoff frequency; (b) keeping w finite and letting $a \to \infty$; in this case the effect of the shield diminishes and we are left with two semi-infinite fins in the unbounded space, which support a zero cutoff frequency mode [27].

Fig. 5 illustrates the possibility to enhance the operating bandwidth $(a(k_c^{21}-k_c^{11}))$ of the circular guide by inserting two asymmetrically placed fins as in Fig. 1. It is clearly seen that, by lowering the cutoff wavenumber of the TE_{11} mode and by raising the corresponding one of the TE_{21} mode, an increase up to 29.3% in bandwidth can be achieved by properly selecting the eccentricity h/a. The bandwidth may be further increased by using two coplanar slots, instead of one. In Fig. 6, the cutoff wavenumbers for the TE_{11} (or TEM) and TE_{21} modes are shown versus $2w/a$ for several widths of the two slots. The solid-line curve, corresponding to the case of two equal slots of width $2w$ joined together, is identical, as expected, to the case of a single slot line of double slot-width $4w$. By properly selecting both slot-width $2w$ and strip-width $2d$ an increase of up to 31.1% in bandwidth can be realized. In the limiting case $w = a - h$, where the double slot line of Fig. 3 goes over to a simple strip line, the corresponding cutoff wavenumbers were found in full agreement with those of [1].

In contrast to the TE modes, the cutoff wavenumbers of the TM^{fin} modes are monotonically decreasing functions of w/a, as seen from Fig. 7. In the limit, as $w/a \rightarrow 1, k_c a$ again approaches with remarkable accuracy the known values for the corresponding modes in the empty guide. Also, contrary to what happens with TE modes, the introduction of symmetrically placed fins causes an increase of bandwidth $(a(k_c^{11} - k_c^{01}))$ of up to 32.2%, as compared with the constant bandwidth value of the simple circular waveguide. Using asymmetrically placed fins causes again an increase in bandwidth, for appropriate combinations of eccentricity and slot-width (Fig. 8); however, this increase never reaches the high values that can be achieved with symmetrical configurations. Therefore, the single centered slot line is the better TM-configuration as far as large bandwidth is concerned.

IV. Conclusions and Generalizations

Efficient singular integral equation techniques have been used to solve for the propagation properties of circularly shielded slot lines. By lowering or raising the cutoff wavenumbers of several modes (via a change in the position and/or the width of the slots) an increase of bandwidth can be achieved.

The analysis presented in this paper covers (or may be readily extended to) a number of related structures. Thus, e.g., one may easily verify that those modes supported by either of the structures of Fig. 1 (for $h = 0$) and Fig. 3, whose $\hat{\rho}$ and/or $\hat{z}$ components of the electric field vanish along the $x = 0$ plane, are the proper ones that can propagate in the single or double ridged semi-circular guides shown in Figs. 9(a)–(b). In addition, single or double slot coupling between two semi-circular guides of different radii, a_1 and a_2, shown in Fig. 10, can be formulated along the lines outlined above. In this way we are led to the S.I.E. (6) or (15) and, also, to the linear algebraic equation (8) (TE-case); in all these equations as well as in (9) and (16) $A_m(a)$ and $f_m(a)$ should be simply replaced by $\frac{1}{2}[A_m(a_1) + A_m(a_2)]$ and $\frac{1}{2}[f_m(a_1) + f_m(a_2)]$, respectively. In the TM-case this procedure leads to the S.ID.E. (20) or (28) and, also, to the algebraic system (22); in these equations as well as in (23) and (29) $B_m(a)$ and $g_m(a)$ should be replaced by $\frac{1}{2}[B_m(a_1)+B_m(a_2)]$ and $\frac{1}{2}[g_m(a_1)+g_m(a_2)]$, respectively.

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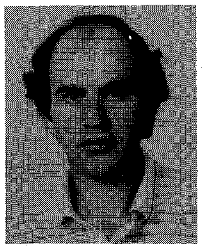

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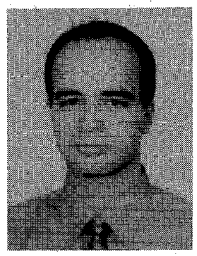

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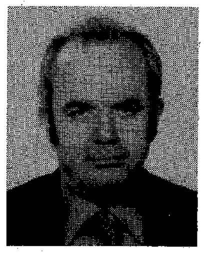

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